

The Design and Implementation of Synthesis Calibration and Imaging in AIPS++

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Abstract. Calibration of and imaging from data measured by synthesis arrays is accomplished in AIPS++ using a very general formulation of interferometry due to Hamaker, Bregman, & Sault (1996). We have designed and implemented a set of C++ classes to provide very flexible capabilities for processing synthesis data from a wide range of nominally different synthesis arrays. We describe the design, the design process, and some details of the implementation. We also comment on the use of AIPS++ for this type of work.

1. Introduction

The processing of data from synthesis arrays has become increasingly sophisticated in recent years, so much so that the final performance of such telescopes is inextricably wedded to the performance of the algorithms used to process the data. Furthermore, some telescopes, such as the proposed Millimeter Array and the Square Kilometer Array, are hard to envisage in operation without the necessary processing algorithms. The drawback to this development is that as the algorithms have increased in sophistication, the difficulty of implementing them in existing packages has increased correspondingly. This difficulty is of course one of the motivations for the AIPS++ Project, in which object-oriented techniques have been adopted with the goal of simplifying development and maintenance of processing algorithms.

AIPS++ is a package for (principally radio) astronomy processing, written in C++ by a collaboration formed between a number of organizations. Those now actively involved are:

- Australia Telescope National Facility,
- Berkeley-Illinois-Maryland Array,
- National Radio Astronomy Observatory, which hosts the Project Center in Socorro, and
- Netherlands Foundation for Research in Astronomy.

AIPS++ is scheduled to undergo a Beta release in early 1997, followed by a limited public release about six months later. More information on AIPS++ may be obtained from the AIPS++ On-Line Documentation¹ page.

The user specifications for AIPS++ were written in late 1991, based upon contributions from a large number of astronomers from the AIPS++ consortium. It is fair to say that the specifications cover all known and some speculative processing algorithms. As well as providing specific functionality in the form of applications, it was also desired that the AIPS++ system provide a means for long-term development of entirely new processing algorithms. The goals for the support of synthesis processing in AIPS++ can be summarized as follows:

- allow all important and, hopefully, most extant calibration and imaging schemes to be coded in AIPS++,
- allow observatories to develop custom calibration and imaging packages,
- allow a programmer to add new calibration and deconvolution algorithms as easily and naturally as possible,
- allow future instruments (such as the NRAO MMA and the SKAI) to be supported in AIPS++,
- support standard observing modes: mosaicked, spectral-line, multi-feed, polarization observations with synthesis arrays and single dishes,
- allow calibration and imaging algorithms to be used as part of higher level algorithms, and
- achieve performance goals that will allow the calibration and imaging system to be used for standard production processing.

Such open-ended flexibility is very demanding of the design of the system, and requires that the abstraction of synthesis telescopes be as general as possible. Thus, ambition drives the level of abstraction higher and higher. However, too high a level of abstraction is difficult to work with and so some compromise must be found. Early attempts to find the correct level of abstraction were not entirely successful. One of the authors advocated an approach based upon the linear mathematics common to all imaging telescopes. A better approach, and one that the Project eventually adopted, is to base the abstraction on the physics of synthesis arrays. In this paper, we present the details of the physical model for synthesis arrays that was adopted. We describe how this was translated into an object-oriented design, and how that design was implemented in AIPS++. We also show some examples of synthesis data processed using this framework. First, we must briefly review synthesis calibration and imaging.

2. Review of Synthesis Calibration and Imaging

Interferometric arrays measure the Fourier transform of the Sky Brightness:

¹<http://aips2.nrao.edu/aips++/docs/html/aips++.html>

$$V(u, v) = \int I(x, y) e^{j2\pi(u \cdot x + v \cdot y)} dx dy \quad (1)$$

In practice, this transform is sampled only at a limited number of discrete points, and it is typically corrupted by antenna-based calibration errors:

$$V_{i,j}(u, v) = g_i g_j^* \int I(x, y) e^{j2\pi(u \cdot x + v \cdot y)} dx dy \quad (2)$$

Calibration and imaging in radio interferometry corresponds to solving for the calibration parameters g and the sky brightness I . There are many special cases where these equations must be modified:

- Wide fields: which cannot be handled as a simple 2-D transform,
- Wide fields: where power receptivity patterns limit the field of view, imposing a need to mosaic, also single dish observations,
- Wide fields: with bandwidth and time-averaging smearing, and
- Polarized radiation: which introduces more terms.

3. The Hamaker-Bregman-Sault Measurement Equation

The simple two-dimensional Fourier transform has been generalized in many different ways. Ultimately, it can be derived from the law for the propagation of mutual coherence in optics and the van Cittert-Zernike equation. Hamaker, Bregman, & Sault (1996) came across one very useful generalization by considering the polarization properties of synthesis arrays.

The required changes to the above equations must take into account the following:

- Each antenna measures two polarizations: either Right and Left Hand Circular or X and Y Linear.
- Typically, the correlator forms all cross correlations, e.g., RR, RL, LR , and LL .
- One often wishes to image Stokes I, Q, U , and V .
- For high quality imaging, it is usually necessary to calibrate, and even self-calibrate, gains and leakages (e.g., R to L and vice versa).

The Hamaker-Bregman-Sault Measurement Equation does take these into account. It is expressed in direct products of 2×2 matrices (Jones Matrices) representing the antenna-based calibration effects, and 4 vectors representing the measured coherence and sky brightness:

$$\vec{V}_{ij} = X_{ij} \left(M_{ij} \left[J^{\text{vis}}_i \otimes J^{\text{vis}}_j^* \right] \sum_k \left[J^{\text{sky}}_i(\vec{\rho}_k) \otimes J^{\text{sky}}_j(\vec{\rho}_k)^* \right] S \vec{I}_k + \vec{A}_{ij} \right) \quad (3)$$

where:

\vec{V} is the 4 vector of coherences, e.g., (RR, RL, LR, LL) ,

X is the non-linear correlator response function,

M represents correlator-based gain errors,

J^{vis} represents antenna-based and non-direction dependent gain effects,

J^{sky} represents antenna-based and direction dependent gain effects (*including the Fourier transform phase factors*),

S is a 4×4 matrix to convert Stokes I, Q, U, V to polarization, e.g., RR, RL, LR, LL ,

\vec{I}_k is the 4 vector of Stokes I, Q, U, V for pixel k , and

\vec{A} is an additive error per coherence sample.

Not surprisingly, this equation does specialize to the usual Fourier transform equation between sky brightness, and furthermore, by the correct choice of the components, such as the J^{sky} , one can derive other imaging modes, such as mosaicking.

For this equation to be useful in handling general problems of calibration and imaging, we need to be able to do the following things:

- evaluate predicted coherences \vec{V}_{ij} if given all terms on right hand side,
- given observed coherences, construct an image of the Sky Brightness, and
- solve for other terms on right hand side given observed coherences and a model for the sky brightness.

Before proceeding further, it is a good idea to split this one equation into separate Sky and Vis Equations:

$$\vec{V}_{ij}^{\text{sky}} = \sum_k \left[J^{\text{sky}}_i(\vec{\rho}_k) \otimes J^{\text{sky}}_j(\vec{\rho}_k)^* \right] S \vec{I}_k \quad (4)$$

$$\vec{V}_{ij} = X_{ij} \left(M_{ij} \left[J^{\text{vis}}_i \otimes J^{\text{vis}}_j^* \right] \vec{V}_{ij}^{\text{sky}} + \vec{A}_{ij} \right) \quad (5)$$

We now also have to make some choice about the class of algorithms that we wish to support. For the moment, we have chosen to limit the main support to those algorithms that can be related to least-squares methods. Perhaps surprisingly, this accommodates many different calibration and imaging algorithms.

For simplicity, consider χ^2 for one time interval:

$$\chi^2 = \sum_{ij} \Delta \vec{V}_{ij}^{*T} \Lambda_{ij} \Delta \vec{V}_{ij} \quad (6)$$

A large class of algorithms, including Maximum Entropy, optimize some combination of χ^2 and another term. For these, one requires the derivatives of χ^2 with respect to the unknown pixels:

$$\frac{\partial \chi^2}{\partial \vec{I}_k} = -2 \Re \left[\sum_{ij} [\mathbf{J}_i(\vec{\rho}_k) \otimes \mathbf{J}_j(\vec{\rho}_k)^*]^{*T} \Lambda_{ij} \Delta \vec{V}_{ij} \right] \quad (7)$$

$$\frac{\partial^2 \chi^2}{\partial \vec{I}_k \partial \vec{I}_k^T} = 2 \Re \left[\sum_{ij} \mathbf{S}^{*T} [\mathbf{J}_i(\vec{\rho}_k) \otimes \mathbf{J}_j(\vec{\rho}_k)^*]^{*T} \Lambda_{ij} [\mathbf{J}_i(\vec{\rho}_k) \otimes \mathbf{J}_j(\vec{\rho}_k)^*] \mathbf{S} \right] \quad (8)$$

Another whole class of algorithms, represented by CLEAN, require a residual image of some sort. A little thought convinces one that a *generalized* residual image can be defined as:

$$\vec{I}_k^D = - \left[\frac{\partial^2 \chi^2}{\partial \vec{I}_k \partial \vec{I}_k^T} \right]^{-1} \frac{\partial \chi^2}{\partial \vec{I}_k} \Big|_{\vec{I}_k=0} \quad (9)$$

The utility of this generalized residual image is that it may be used in many iterative-update type algorithms in which a trial estimate image is updated from the residual image.

Note that the corresponding PSF may well not be shift-invariant, and so either some approximation must be used or the algorithm must accommodate a shift-variant PSF.

Many *calibration* algorithms require derivatives of χ^2 with respect to the various gain matrices. As an example, consider the case where only one term is required:

$$\frac{\partial \chi^2}{\partial \mathbf{G}_{i,p,q}} = -2 \sum_j [\mathbf{U}_{p,q} \otimes \mathbf{G}_j^*]^{*T} \Lambda_{ij} \Delta \vec{V}_{ij} \quad (10)$$

$$\frac{\partial^2 \chi^2}{\partial \mathbf{G}_{i,p,q} \partial \mathbf{G}_{i,p,q}^{*T}} = 2 \sum_j [\mathbf{U}_{p,q} \otimes \mathbf{G}_j^*]^{*T} \Lambda_{ij} [\mathbf{U}_{p,q} \otimes \mathbf{G}_j^*] \quad (11)$$

where the matrix $\mathbf{U}_{p,q}$ is unity for element (p, q) and zero otherwise.

These gradients may be used in a least squares solution for the calibration parameters.

In an object-oriented system, one would want to provide services to evaluate these gradients (and the original equations) for quite general forms of the components of the equations. The interface of the components need only be specified as far as it concerns the calculation of these quantities. Otherwise, the behavior of the components is unspecified, and may be varied to suit different contexts.

A consumer of these services has to be responsible for the model or trial estimates but not for evaluating χ^2 and associated gradients.

4. C++ Classes

The support for the Hamaker-Bregman-Sault Measurement Equation is provided by a number of C++ classes:

SkyEquation and **VisEquation** are concrete classes responsible for evaluating these equations, χ^2 , and the gradients of χ^2 ,

VisSet is a concrete class that provides coherence data to the **SkyEquation** and **VisEquation** classes and stores the results of prediction and correction,

SkyModel supplies a set of images such as \vec{I} to the **SkyEquation** class,

SkyJones supplies sky-plane-based calibration effects to the **SkyEquation** class by multiplying a given image by matrices, such as J^{sky} ,

FTCoh and **IFTCoh** perform forward and inverse Fourier transforms,

VisJones applies visibility-plane-based calibration effects via either 2×2 or 4×4 matrices such as J^{vis} ,

MIfr applies interferometer-based gain effects **VisEquation** via a 4×4 matrix M ,

ACoh applies interferometer-based offsets via a 4 vector \vec{A} , and

XCorr applies a non-linear correlator function via a function X to a 4 vector.

The *MeasurementComponents* **SkyModel**, **SkyJones**, **VisJones**, **MIfr**, **ACoh**, and **XCorr** can solve for themselves given a specific **SkyEquation** or **VisEquation** and **VisSet**. Programmers write special versions of these *MeasurementComponents* for particular circumstances. As a concrete example, consider the calibration of a phase screen across a compact array such as the proposed Millimeter Array. In conventional self-calibration procedures, one would solve for the phases of all 40 antennas. A more sensible and robust approach is to solve only for a limited number of parameters describing a phase screen across the array. This could be simply a tilted screen or perhaps a few curvature terms, but the main point is that since the number of degrees of freedom would be limited, the robustness and sensitivity would both improve. In the AIPS++ approach, the programmer merely has to develop a *MeasurementComponent* that uses the **VisEquation** services to get gradients of χ^2 for the relevant Jones matrices, and converts this gradient information into update information for parameters of the phase screen. The interface to the **VisEquation** is thus fixed, but the internal behavior of the component is unspecified.

A similar approach works for the **SkyModel**. Here all the gradients are calculated in terms of a discrete pixellated images, but any given **SkyModel** may use these gradients as desired, perhaps to drive the solution for the parameters of Gaussian components.

Simulation is straightforwardly accommodated by providing *MeasurementComponents* that generate, rather than solve for, instrumental effects.

5. Applications Level Code

The design of the applications level code has been through a number of revisions. We initially used a monolithic, command-line program with lots of switches to control behavior—the prime advantage of this approach is simplicity: one could exercise all of the code this way. For the end-user, it has one prime drawback:

the behavior of the application is controlled by a number of switches and it is hard to understand and document. Despite these shortcomings, it is still the best way to write a test program.

With this in mind, we changed the design to be based upon objects that could be directly invoked from the AIPS++ CLI, Glish, using the AIPS++ Distributed Object mechanism. By decomposing the code from the monolithic applications, we derived objects with different responsibilities, all coded in terms of the underlying C++ classes. The five user-level objects were:

calibrator which reads, writes, interpolates, applies, and solves for calibration information, e.g., to correct a MeasurementSet for parallactic angle:

```
calibrator:=imager.calibrator();
calibrator.initialize(calibrator, '3C273XC1.MS');
calibrator.set(calibrator, 'P', 30.0);
calibrator.correct(calibrator);
calibrator.write(calibrator);
```

flagger which flags data using a number of methods,

imagemaker which makes an empty template image,

imagesolver which solves for an image (i.e., deconvolves using, e.g., CLEAN or Dan Brigg's NNLS method), and

weighter which applies weighting to a dataset.

In the final revision of the application code, we watched what people did and then built standard sequences of object invocations into CLI functions. This is the **imager** package in AIPS++. A typical example of the use of the **imager** would be as follows:

```
include "imager.g"           # Include definition of imager and di
di.initialize("3C273XC1")    # Initialize inputs
di.getfits()                 # Get data from the FITS file
di.show()                    # Show the inputs
di.cell:=0.7                 # Cellsize
di.makeimage()               # Make an empty template image
di.P.t:=60.0                 # Time scale for parallactic angle
di.G.t:=300.0                # Time scale for antenna gain changes
di.D.t:=3600.0               # Time scale for polarization leakage changes
di.initcal()                 # Set up calibration objects
di.beam:=[2.5, 2.5, 0]       # Beam size
di.clean.niter:=10000        # Number of clean iterations
di.deconv()                  # Perform clean deconvolution
di.dis()                     # Display the image
for (selfcal in 1:5)         # Perform 5 loops of....
{
    di.scal()                # Perform one selfcal iteration
    di.plotres()              # Plot residual visibility
    di.dis()                  # Display the image
}
```

The object **di** is the default instance of the class **imager**. Our next planned step is to build a GUI interface that invokes these functions and integrates display and plotting.

6. Comments on Design, Implementation, and Developing in AIPS++

We performed a detailed analysis of the Hamaker-Bregman-Sault formalism before starting the design, and read a lot. But in the end, we did not adopt any formal methodology or diagramming tools. We did lots of prototype coding, all in C++, although next time, we would probably do at least the early stages in Glish. We split responsibilities for development in different areas, and reconciled code divergences, by hand, every few days. The overall breakup into classes was settled fairly early on and changed relatively little. The assignment of responsibilities to classes changed a lot as we looked for and found a natural split of the evaluation of gradients of χ^2 . This probably represented most of the experimentation that was performed. Symmetry in the design between the Jones matrices and the `SkyModel` was the most powerful organizing principle that we came upon. Enforcing uniformity of interface across objects was also important. Finally, we came to the split between `SkyEquation` and `VisEquation` quite late on. While the split was obvious, the reasons for making the design split became more pressing as we got deeper into the details and began to think in more detail about, e.g., Single Dish processing.

Clearly this is not a simple waterfall design process, in which specification leads to analysis, design, implementation, testing, and deployment. Instead it is probably better described in terms of the spiral model whereby one revisits each of the above stages a number of different times, each time having progressed from the last time. The design is still evolving, somewhat, as we try to optimize performance, especially on spectral line datasets. We think that such a spiral design process is mandated by the complexity of the problem that we were trying to solve.

We wrote special classes for matrices with few elements and special symmetries (`SquareMatrix`) and vectors with few elements (`RigidVector`). It was necessary to drop into FORTRAN for some inner loops, e.g., FFTs, gridding, and CLEAN loops.

Our conclusions on developing within AIPS++ are as follows. AIPS++ is (currently) very complex but is well-featured. It seems that C++ is a net win but the buy-in is large (6–12 months for competence to do the type of design work presented here). There are many new concepts that one must learn (e.g., iterators) that simply cannot be avoided. Compilers are an obstacle: *Gnu* is the standard compiler and is the best of the lot. Finally, we found the loose collection of AIPS++ tools united by Glish to be wonderful for development.

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References

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