

Image Reconstruction with Few Strip-Integrated Projections: Enhancements by Application of Versions of the CLEAN Algorithm

M. I. Agafonov

*Radiophysical Research Institute (NIRFI), 25 B. Pecherskaya st.,
Nizhny Novgorod, 603600, Russia, E-mail: agfn@nirfi.nnov.su*

Abstract. Iterative algorithms with non-linear constraints are very attractive in image reconstruction with only a few strip-integrated projections. We present research into various versions of the iterative CLEAN algorithm for the solution of this problem. We suggest a method to determine the area of permissible solutions in complicated cases for two CLEAN algorithms. This procedure was named 2-CLEAN Determination of Solution Area (2-CLEAN DSA).

1. Introduction

Two dimensional image reconstruction from 1-D projections is often hampered by the small number of available projections, by an irregular distribution of position angles, and by position angles that span a range smaller than about 100° . These limitations are typical of both lunar occultations of celestial sources and observations with the fan beam of a radio interferometer, and also apply to greatly foreshortened reconstructive tomography.

2. Deconvolution Problem

The problem requires the solution of the equation

$$G = H * F (+noise) , \quad (1)$$

where $F(x, y)$ is the object brightness distribution, $H(x, y)$ is the fan (dirty) beam, and $G(x, y)$ is the dirty (summary) image. The classical case (Bracewell & Riddle 1967) needs a number of projections $N \geq \pi D/\varphi$, where φ is the desired angular resolution, and D is the diameter of the object. The incomplete sampling of $H(u, v)$ requires the extrapolation of the solution of $F'(x, y)$ using non-linear processing methods.

2.1. The Iterative Algorithms

The general scheme of an iterative algorithm is

$$F^{k+1} = r_\alpha C F^k + \lambda (G - H C F^k) , \quad (2)$$

where λ is the loop gain ($0 < \lambda < 2/\max H(u, v)$), $C = C_1 C_2 \dots C_n$ are the limitations, and r_α is the stabilizer.

The simple standard CLEAN (Högbom 1974) is the best known realization of iteration schemes in radio astronomy. But the algorithm has defects (stripes and ridges) in areas of extended emission. CLEAN was used for the image reconstruction of the Crab Nebula from four lunar occultation profiles (Maloney & Gottesman 1979; Agafonov et al. 1986). However, more complete information is needed for an extended object. Trim Contour CLEAN (TC-CLEAN) (Steer et al. 1984) gave hope for the improvement of image quality with extended features, but it needed a study in different practical cases (Cornwell 1988).

2.2. Numerical Modeling

The process of solution convergence by σ (ERROR of initial and control 1-D profiles) minimization with variation of parameters λ and TC (Trim Contour level) was investigated (Agafonov & Podvojskaya 1989; Agafonov & Podvojskaya 1990) for both algorithms using of the following procedure:

- **2-D object model** \rightarrow **1-D profiles** \rightarrow **Dirty image**
- **CLEAN** (λ) or **TC-CLEAN** (λ, TC) using the **Dirty beam**
- **Control test** from clean maps: **Calculation of σ** (ERROR of control and initial 1-D profiles)
- **Correction of λ or λ, TC to $\min \sigma$**

The process of parameter (λ or λ, TC) optimization to $\min \sigma$ was shown very well graphically (Agafonov & Podvojskaya 1989).

CLEAN The dirty map peak is the target of each iteration. The choice of loop gain λ was not clear in the original scheme, but it significantly influences the solution. For example, by modeling the test object (Crab Nebula map at 1.4 GHz) the optimum range of λ was found to be about 0.05–0.10 (from the dependence of $\sigma(\lambda)$ in this map). But the optimum value depends on the object structure. The algorithm has high instability for distributed objects. Changes in the resulting maps as a function of σ with small changes of λ testifies to this instability. We attempted to increase stability by: (i) choosing solutions with $\min \sigma$ from the optimum interval and then averaging; and (ii) by special processing—such as a complex method like speckle-masking (but increasing the computational efforts).

TC-CLEAN A smooth function is subtracted at each iteration. Trim contour (TC) is used for the choice of components per iteration. TC must be low, but above the level corresponding to the true object dimensions. The algorithm has high stability, a simple choice of λ and TC , converges in few iterations, and is computationally efficient.

3. Discussion and Conclusions

A simple object (consisting of the peaks) may be successfully restored by the standard CLEAN. The results obtained by both methods are practically identical for a simple object consisting of individual components, but TC-CLEAN is more computationally efficient. A difference between the solutions is observed

for complicated objects with small components in areas of extended emission. The standard CLEAN reconstruction has a “grooved” structure for such areas.

For smoothed 1-D profiles with small “hillocks,” the solution can be obtained from the isolated individual components (CLEAN), and also from the more smoothed components (TC-CLEAN) by using the same $\min \sigma$ for the initial and control profiles. CLEAN increases the contrast of small components, but the extended background decreases because of the “grooves.” If $\min \sigma$ (CLEAN) $\cong \min \sigma$ (TC-CLEAN), the solutions will be formally equivalent for both algorithms, and so we have two choices: (i) to prefer the result corresponding to the physical peculiarities of the object in accordance with *a priori* information; or (ii) to assume the existence of a probable class of solutions between the “obtuse” (smooth) one from TC-CLEAN and the “sharp” one from CLEAN.

CLEAN forms the solution from the sum of peaks, and the result is the sharpest variant permissible within the established constraints. On the other hand, TC-CLEAN accumulates its result from the most extended components that satisfy the constraints, producing the smoothest solution (Agafonov & Podvojskaya 1990).

We carried out the study of TC-CLEAN applied to image reconstruction with few projections. TC-CLEAN proved to be an effective and stable solution. CLEAN emphasizes only individual features and needs a special treatment to obtain the solution stability (except the object from the peaks). We suggest determining the area of permissible solutions of complicated objects with the help of both algorithms. This procedure was named 2-CLEAN Determination of Solution Area (2-CLEAN DSA). It is also useful to study the possibility of reconstruction (the reality of the components on the map) for any new case with poor *UV*-filling by using a similar method used for our observational test object. The 2-CLEAN DSA procedure can show, in complicated cases, a range of possible images from “obtuse” to “sharp” variants satisfying imposed constraints and poor *a priori* information.

4. Maps from Real Lunar Occultation Observations

In certain cases, usually at lower frequencies, the angular resolution of synthesis radio telescopes is insufficient. However, observations of an object during lunar occultations provide 1-D brightness profiles with high angular resolution. This is also useful in observations with optical instruments. The lunar limb is approximated by a plane screen, moving through different position angles. The first images of the Crab Nebula from four projections of lunar occultations was presented by using the standard CLEAN (Maloney & Gottesman 1979; Agafonov et al. 1986). But the maps had defects in extended areas due to the reconstruction algorithm. The application of TC-CLEAN was used in the reconstruction of the Crab Nebula map at 750 MHz with angular resolution $20 \times 35''$ (Agafonov et al. 1990). The 1-D profiles were obtained by observations using the 70 meter dish (RT-70) in West Crimea. The method of 2-CLEAN DSA allowed us to determine that the area of permissible solutions lies formally between the “sharp” (CLEAN) and “smooth” (TC-CLEAN) variants. The two maps were generally similar. The standard CLEAN increased the contrast of small components, while the TC-CLEAN map gave a better agreement with known *a priori* information about the Nebula, and was closer to the true brightness distribution of

the Crab. The CLEAN variant of the map can be used only for information about the location of the small components.

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