Astronomical Data Analysis Software and Systems XIII
ASP Conference Series, Vol. 314, 2004
F. Ochsenbein, M. Allen, and D. Egret, eds.

Boxiness estimation method with fourth order moments

Masafumi Yagi

National Astronomical Observatory of Japan, Mitaka, Tokyo, 181-8588, Japan

Abstract. I present a method to estimate fourth order cosine coefficient of Fourier harmonics, so called boxiness, from second and fourth order moments in Cartesian coordinate. The method is faster than traditional fitting method about by order and have enough accuracy for $r > 10pix$ objects.

1. Introduction

Boxiness, the fourth order cosine coefficient of Fourier harmonics, is known to be an important photometric parameters in study of early type galaxies (e.g. Kormendy & Djorgovski 1989) The boxiness parameter is, however, not measured in recent huge survey data. One of the reason would be the fact that standard boxiness measurement process is complicated and takes time; fit ellipse, take residual, iterate if needed, and fit trigonometric functions. Another reason could be that fitting method is usually not full-automatic, and requires manual interactive operation, which disables us from constructing pipeline analysis.

About 2nd-order Fourier components, such as major/minor axis, many surface photometry packages estimate the parameters from 2nd-order Cartesian moments (e.g. Stobie 1980), and therefore automatic measurement is implemented. Extending the idea, I developed a procedure to estimate the boxiness from 2nd and 4th order non-weighted Cartesian moments.

2. Method

In this study, I follow the formalization of the general ellipse by Stobie (1980); setting original coordinate as (x,y), and coordinate along the major and minor axis of the best fit ellipse as (X,Y),

$$X = xcos(\phi) - ysin(\phi) - x0 \quad (1)$$
$$Y = xsin(\phi) + ycos(\phi) - y0, \quad (2)$$

where x0, and y0 are the center of the image, and ϕ is the rotation angle. The shape of isophote is then supposed to be expanded with Fourier components as

$$X^2 + \frac{Y^2}{Q^2} \leq a^2(1 + \sum c_n cosn(\theta + \omega_n))^2, \quad (3)$$

where $\theta = tan^{-1}((Y/b)/(X/a))$, a and b are semi-major axis and semi-minor axis, and Q is the axis ratio ($Q = b/a$). With this notation, $a_4 = c_4 cos4\omega_4$ is the boxiness-parameter. It corresponds to B_4/r in Carter (1978), $a(4)/a\sqrt{Q}$ in Bender et al. (1988), and c_4 in Milvang-Jensen & Jørgensen (1999). If a_4 is positive, the isophote is disky, and if a_4 is negative, the isophote is boxy. I neglect 3rd and higher than 4th order components for simplicity, for they are not dominant terms in elliptical galaxies. The isophote shape is thus written as

$$X^2 + \frac{Y^2}{Q^2} \leq a^2(1 + c_4 cos4(\theta + \omega_4))^2 \tag{4}$$

I define a moment of a shape as non-weighted, $< z >= \int zdA/\int dA$. With such moment, estimated major/minor axis can be written as

$$\begin{aligned} a_0 &= \sqrt{2(p+q)} \\ b_0 &= \sqrt{2(p-q)}, \end{aligned}$$

where p and q are Stokes parameters,

$$\begin{aligned} p &\equiv < x^2 > + < y^2 > \\ q^2 &\equiv (< x^2 > - < y^2 >)^2 + 4 < xy >^2 \end{aligned} \tag{5}$$

(Stobie 1980). It should be noted that p and q are rotation invariant combination of moments. Such rotation invariant combination is obtained by integrating $r(cos^2 n\theta + sin^2 n\theta)$. As the boxiness is also rotation invariant, it is supposed to be estimated from such combinations. For fourth order case, rotation invariant combinations are

$$\begin{aligned} p_4 &\equiv < x^4 > +2 < x^2y^2 > + < y^4 > \\ q_4^2 &\equiv (< x^4 > + < y^4 > -6 < x^2y^2 >)^2 + 16(< x^3y > - < xy^3 >)^2. \end{aligned} \tag{6}$$

As a_4 is usually less than 10% (e.g. Bender et al. 1988), one can neglect c_4^2 and approximate as $1 + O(c_4^2) \sim 1$. With this approximation, p_4 of Eqn. (4) is written as

$$\begin{aligned} p_4 &= < x^4 > +2 < x^2y^2 > + < y^4 > \\ &\sim \frac{\pi Q a_0^6}{8A} \times \\ &\quad \left((1 + Q^4 + 2/3Q^2) + a_4(1 + Q^4 - 2Q^2)\right), \end{aligned} \tag{7}$$

where A is the total area of the isophote. We can easily solve equation to estimate a_4 as a function of p_4, Q, a_0, and A,

$$a_4 \sim \frac{1}{(1-Q^2)^2}\left(\frac{8Ap_4}{Q\pi a_0^6} - a_0^4(1 + Q^4 + 2/3Q^2)\right). \tag{8}$$

3. Implementation and Simulation

The Eqn. 7 is implemented in standard C as a code for measuring boxiness profile, $a_4(r)$. Rapid flooding method (Treuenfels 1994) is adopted for extracting isophotal shape profile. And in moment calculation, "single visit with temporal mean method" is adopted, which uses recurrence formulae; $< x_n^m >= f(x_n, < x_{n-1} >, < x_{n-1}^2 >, ... < x_{n-1}^m >)$. As rapid flooding method visit once for each pixel, and single visit with temporal mean method can calculate moment by the single visit, the combination succeeds in quick boxiness estimation. For boxiness estimation only, the code is about 20 times faster than IRAF/STSDAS ellipse task (Busko 1996) tested on Solaris/SPARC architecture.

The error is estimated by applying the code to simulated images. For large ($r = 400$) images, relative error of a_4 is always less than 10%, with no systematic error with rotation angle. All except rounder (Q=0.9) image have absolute error less than 0.001. For smaller images, error correlates with $r = \sqrt{ab} = a\sqrt{Q}$. Error is smaller than 0.005(rms) for $r > 15$ pix, and smaller than 0.01(rms) for $r > 10$ pix.

4. Result and Future Plans

The implementation is applied to real image of 23 E/S0 galaxies taken from Frei (1996) R-band data. Our result is consistent with other studies within relative error of 10% or absolute error of 0.01. The accuracy is as expected from model analysis, and enough for boxy/disky discrimination. Figure 1 shows an example of the result. Some inconsistency is found around log(a)∼1.8, where star debris is seen in the image. It could be the difference of the image quality, preprocessing of the image, or post-processing of the boxiness profile.

As a next step, I plan to investigate flux-weighted moments, preprocessing (e.g. Unsharpmasking) of images, and interpolation of internal parts or interpolation of moments at small radii.

References

Bender, R., Döbereiner, S. & Möllenhoff, C. 1988, A&AS, 74, 385

Busko, I. 1996, in ASP Conf. Ser., Vol. 101, ADASS V, ed. G. H. Jacoby & J. Barnes (San Francisco: ASP), 139

Frei, Z., Guhathakurta, P., Gunn, J. E. & Tyson, J. A. 1996, AJ, 111, 174

Jedrzejewski, R. I. 1987, MNRAS, 226, 747

Kormendy, J. & Djorgovski, S. 1989, ARA&A, 27 235

Milvang-Jensen, B. & Jørgensen, I. 1999, BaltA, 8, 535

Michard, R. & Marchal, J. 1994, A&AS, 105, 481

Peletier, R. F., Davies, R. L., Illingworth, G. D., Davies, L. E. & Cawson, M. 1990, AJ, 100, 1091

Stobie, R. S. 1980, JBIS, 33, 323

Treuenfels, A. 1994, C Users Journal, 12, 8, 39

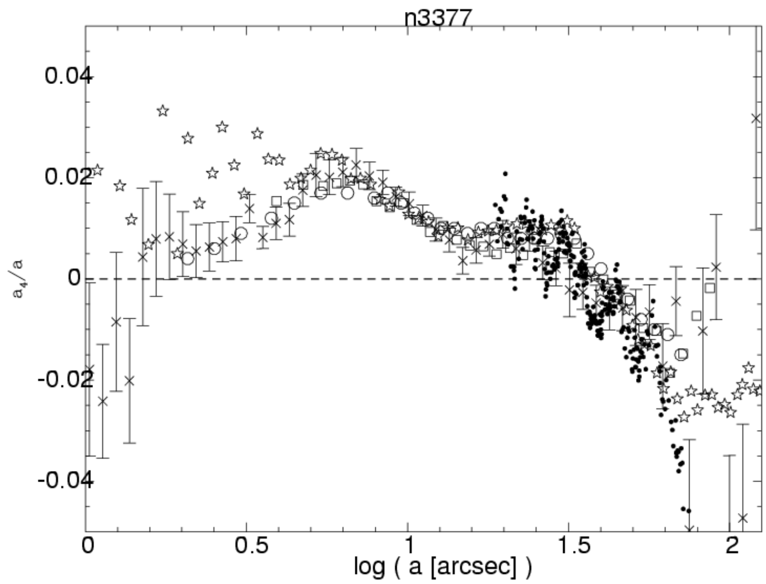

Figure 1. An example of boxiness estimation. Comparison of boxiness profiles. Our estimation is shown as black dots. Measurement with IRAF/STSDAS/ellipse (Busko 1996) is shown as cross with error bars. Other symbols are taken from literatures. Open circles, open squares, and open stars represent measurement by Jedrzejewski (1987), Peletier et al. (1990), and Michard & Marchal (1994), respectively.

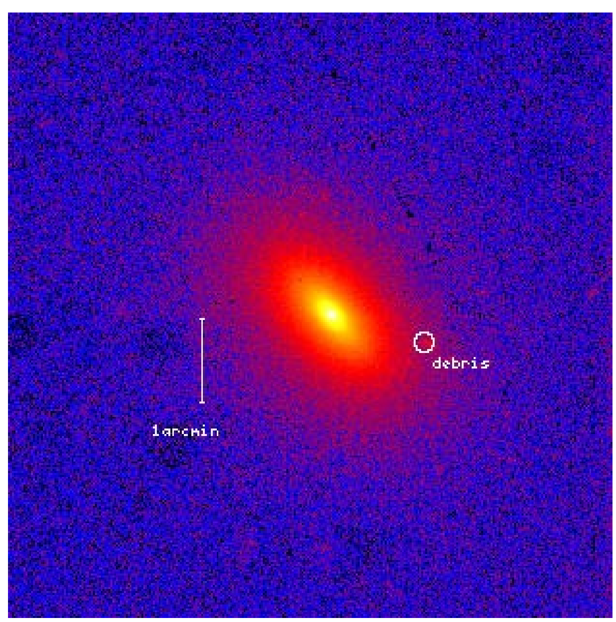

Figure 2. R-band image of NGC 3377 from Frei (1996), The debris of star removal affect the boxiness at log(a)∼1.8 in Fig 1.