Astronomical Data Analysis Software and Systems XII
ASP Conference Series, Vol. 295, 2003
H. E. Payne, R. I. Jedrzejewski, and R. N. Hook, eds.

Generalized Self-Calibration for Space VLBI Image Reconstruction

Sergey F. Likhachev

Astro Space Center, Profsojuznaya 84/32, GSP-7, 117997, Moscow, Russia

Abstract. Generalized self-calibration (GSC) algorithm as a solution of a non-linear optimization problem is considered. The algorithm allows to work easily with the first and the second derivatives of a visibility function phases and amplitudes. This approach is important for high orbiting Space VLBI data processing. The implementation of the GSC-algorithm for radio astronomy image restoration is shown. The comparison with other self-calibration algorithms is demonstrated. The GSC-algorithm was implemented in the radio astronomy imaging software project Astro Space Locator (ASL) for Windows developed at the Astro Space Center.

1. Image reconstruction: Basic concepts

Definition 1.1. A space of complex functions $V(\mathbf{u})$ dual to the brightness distribution $I(\mathbf{x})$ and risen by an operator F, is called a space of *visibility functions* or *spatial coherency.*

$$V(\mathbf{u}) = F\left\{\mathbf{I}(\mathbf{x})\right\} = \int\limits_{-\infty}^{\infty} \mathbf{I}(\mathbf{x}) \exp\left\{-2\pi i \left\langle \mathbf{x}, \mathbf{u} \right\rangle\right\} d\mathbf{x} \qquad (1)$$

Definition 1.2. Let us define *an object* as a domain of the Universe that is a subject of the investigation whose brightness distribution could be represented as a 2-D function with infinite spatial frequency spectra.

Definition 1.3. Let us define *an image* of the object as a result of creation *by unknown spatial brightness distribution*

$$\mathbf{I}_{true}(\mathbf{x})$$

(the object) of *an illumination distribution*

$$\mathbf{J}_{true}(\mathbf{x}),$$

i.e.,

$$\mathbf{J}_{true}(\mathbf{x}) = (H\mathbf{I}_{true})(\mathbf{x}).$$

In other words, we have an original object located somewhere in the space (Universe) and we can observe only some projection of this object on this space. For example, one of the projections of the object can be its electromagnetic emission of the object in a given spectral band and in a given moment of time.

2. VLBI Image Restoration as an Approximation Procedure

Let us consider a metrics

$$\rho = \sum_{ij} w_{ij} \left[V_{ij} - \hat{V}_{ij} \right]^2 \Rightarrow \min \tag{2}$$

where V_{ij} is a measurements of the visibility function and $\hat{V}_{ij}$ is an approximating function.

There exist a few possible approximating functions:

1. Orthogonal approximation:

$$\hat{V}_{ij} = \sum_{ij} c_{ij} \varphi_{ij}$$

If φ_{ij} is a Fourier basis, the orthogonal approximation is known in VLBI as a CLEAN algorithm.

2. Bi-orthogonal approximation:

$$\hat{\mathbf{V}} = \mathbf{g} \cdot \psi \cdot \bar{\mathbf{g}}$$

If ψ_{ij} is a 2-D complex function (a model), the bi-orthogonal approximation is known in VLBI as a self-calibration algorithm.

3. Non-parametrical approximation:

$$\sum_i \lg I_i - \lambda \cdot \rho \max$$

(known in VLBI as a MEM algorithm).

4. Mathematical programming approximation:

$$\mathbf{D} = \mathbf{B} \cdot \mathbf{I}$$

$$\mathbf{I} \geq 0$$

(known in VLBI as a so-called NNLS algorithm).

3. Generalized Self-Calibration (GSC) as a Problem of Non-linear Optimization

Let us consider an expression

$$\mathbf{V}_{t\nu} = \mathbf{g}_{t\nu} \mathbf{V}_{t\nu}^{true} \bar{\mathbf{g}}_{t\nu} + \varepsilon_{t\nu} \tag{3}$$

where $\mathbf{g}_{t\nu} = diag\,[g_i]$, $\dim\{\mathbf{g}_{t\nu}\} = [N-1 \times N-1]$, $\mathbf{V}_{t\nu} = \{V_{ijkl}\}$ visibility matrix was measured on the baseline (i,j) for a given moment of time t_k and frequency ν_l, $\dim\{\mathbf{V}_{t\nu}\} = [N-1 \times N-1]$, $\mathbf{V}_{t\nu}^{true}$ true visibility function for the baseline (i,j), for a given moment of time t_k and frequency ν_l, $\dim\{\mathbf{V}_{t\nu}\} = [N-1 \times N-1]$, $\varepsilon_{t\nu}$ additive noise.

Let us consider a discrepancy

$$\mathbf{z}_{t\nu} = \mathbf{g}_{t\nu}\hat{\mathbf{V}}_{t\nu}\bar{\mathbf{g}}_{t\nu} - \mathbf{V}_{t\nu} \tag{4}$$

It is necessary to obtain:

$$\arg\min_{g_i} \rho = \|\mathbf{z}_{t\nu}\|^2 \tag{5}$$

where $\hat{\mathbf{V}}_{t\nu}$ is a model of $\mathbf{V}_{t\nu}^{true}$, $\hat{\mathbf{V}}_{t\nu}$ is upper triangular matrix with $\hat{V}_{ij} = 0$. The solution $\mathbf{g}_{t\nu}$ obtained on the basis generalized Newton's algorithm with pseudo-inversion.

4. Local Approximation of Gains

Let us represent *a complex function* $g_i(t', \nu')$ as a time series in the neighborhood of a point (t'_0, ν'_0). Then

$$\begin{aligned} g_i(t'_0 + \Delta t, \nu'_0 + \Delta\nu) &= g_i(t_0, \nu_0) + \frac{\partial g_i(t_0, \nu_0)}{\partial t'}\Delta t + \frac{\partial g_i(t_0, \nu_0)}{\partial \nu'}\Delta\nu + \\ & O\left(\Delta t^2 + \Delta\nu^2\right) \end{aligned} \tag{6}$$

Let us introduce the following notations:

- let us call
$$r_i = \frac{\partial \varphi_i(t_0, \nu_0)}{\partial t}$$
as *a fringe rate*;
- let us call
$$\tau_i = \frac{\partial \varphi_i(t_0, \nu_0)}{\partial \nu}$$
as *a fringe delay.*

Both values are complex ones and can be represented as

$$\begin{aligned} & g_i(t'_0 + \Delta t, \nu'_0 + \Delta\nu) = \\ & \quad \left\{\left[a_i(t_0, \nu_0) + \frac{\partial a_i(t_0, \nu_0)}{\partial t'}\Delta t + \frac{\partial a_i(t_0, \nu_0)}{\partial \nu'}\Delta\nu\right] + \right. \\ & \quad i \cdot [a_i(t_0, \nu_0)\, r_i \Delta t + a_i(t_0, \nu_0)\, \tau \Delta\nu]\} \times \exp\{i \cdot \varphi_i(t_0, \nu_0)\} + \\ & \quad O\left(\Delta t^2 + \Delta\nu^2\right) \end{aligned} \tag{7}$$

Example. If $a_i(t_0, \nu_0) = const$ (no amplitude calibration) then

$$\begin{aligned} & g_i(t'_0 + \Delta t, \nu'_0 + \Delta\nu) = \\ & \quad \{const \cdot [r_i \Delta t + \tau\Delta\nu]\} \times \exp\left\{i \cdot \left[\varphi_i(t_0, \nu_0) + \frac{\pi}{2} + 2k\pi\right]\right\} + \\ & \quad O\left(\Delta t^2 + \Delta\nu^2\right) \end{aligned} \tag{8}$$

and obtain a well-known expression for phase calibration (see Schwab 1981).

A value $O\left(\Delta t^2 + \Delta\nu^2\right)$ describes derivatives of the second order that is necessary to take into account for Space VLBI imaging.

5. Some Imaging Problems for High Orbiting Space VLBI

Definition 5.1. If for any three radio telescopes

1. there exists its closing, i.e.,

$$\vec{\varphi}_1 + \vec{\varphi}_2 + \vec{\varphi}_3 = 0;$$

2. any two baselines

$$\|\mathbf{b}_{13}\| \,\&\, \|\mathbf{b}_{23}\| > D_{Earth},$$

3. and

$$\|\mathbf{b}_{23} - \mathbf{b}_{13}\| / \|\mathbf{b}_{12}\| \to 1,$$

then the VLBI can be called *high orbiting space VLBI*.

In case of a High Orbiting SVLB mission a good (u,v)-coverage does not guarantee high quality images because

$$\Delta\varphi_{12} \approx \delta\varphi_{rms}$$

is an "apogee phase gap."

6. Implementation of GSC in the ASL for Windows

The software project, Astro Space Locator (ASL) for Windows 9x/NT/2000 (code name ASL_Spider 1.0) is developed by the Laboratory for Mathematical Methods[1] of the ASC to provide a free software package for VLBI data processing. We used the Microsoft Windows NT/2000 and MS Visual C++ 6.0 on IBM compatible PCs as the platform from which to make data processing and reconstruction of VLBI images.

7. Outcomes

A generalized self-calibration (GSC) algorithm was developed. The solution was obtained as a non-linear optimization in the Hilbert space L_2. GCS describes not only the first derivatives but also of the second derivatives that is necessary to take into account for Space VLBI imaging. A global fringe fitting procedure is just an initialization (zero iteration) of GSC. GSC allows to obtain more stable and reliable results than traditional self-calibration algorithms.

References

Schwab, F. R. 1981, VLA Scientific Memorandum, No. 136, NRAO

[1] `http://platon.asc.rssi.ru/dpd/asl/asl.html`