Astronomical Data Analysis Software and Systems IX
ASP Conference Series, Vol. 216, 2000
N. Manset, C. Veillet, and D. Crabtree, eds.

Recent Advances in Parameter Estimation in Astronomy with Poisson-Distributed Data

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Abstract. Applying the standard weighted mean formula, $[\sum_i n_i \sigma_i^{-2}]/[\sum_i \sigma_i^{-2}]$, to determine the weighted mean of data, n_i, drawn from a Poisson distribution, will, on average, underestimate the true mean by $\sim$1 for all true mean values larger than $\sim$3 when the common assumption is made that the error of the ith observation is $\sigma_i = \max(\sqrt{n_i}, 1)$. This small, but statistically significant offset, explains the long-known observation that chi-square minimization techniques using the modified Neyman's χ^2 statistic, $\chi^2_{\rm N} \equiv \sum_i (n_i - y_i)^2 / \max(n_i, 1)$, to analyze Poisson-distributed data will typically predict a total number of counts that underestimates the true total by about 1 count per bin. Based on my finding that the weighted mean of data drawn from a Poisson distribution can be determined using the formula $[\sum_i [n_i + \min(n_i, 1)](n_i + 1)^{-1}]/[\sum_i (n_i + 1)^{-1}]$, I have proposed a new χ^2 statistic, $\chi^2_\gamma \equiv \sum_i [n_i + \min(n_i, 1) - y_i]^2/[n_i + 1]$, should always be used to analyze Poisson-distributed data in preference to the modified Neyman's χ^2 statistic (Mighell 1999, ApJ, 518, 380). I demonstrated the power and usefulness of χ^2_γ minimization by using two statistical fitting techniques and three χ^2 statistics to analyze simulated X-ray power-law 15-channel spectra with large and small counts per bin. I showed that χ^2_γ minimization with the Levenberg-Marquardt or Powell's method can produce excellent results (mean errors $\lesssim$3%) with spectra having as few as 25 total counts.