Astronomical Data Analysis Software and Systems VIII
ASP Conference Series, Vol. 172, 1999
D. M. Mehringer, R. L. Plante, and D. A. Roberts, eds.

A Method to Test the Adequacy of a Model to Observations

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Abstract. It is well known that the Least Squares method returns the values of the parameters regardless the correspondence of the model to observations. If the model does not correspond to the data one obtains the formal values of the parameters, which have no physical sense. Nevertheless, many people use values thus obtained in their theories and practice.

The proposed method based on the orthogonal representation of the equations of a model is free of this fault since it tests the consistency of the model and data.

This method have been used to find out the compatibility of the standard kinematical model with the proper motions of stars in the Hipparcos catalogue. It shows that there are some difficulties with the standard model of the Galactic rotation, and, on the contrary, there is a good agreement with the model of the Sun motion.

1. Traditional Way to Study Stellar Kinematics

A common practice in astronomy is: gathering data, constructing a model, evaluating of the parameters of the model by the Least Squares technique.

Let us take an example from stellar kinematics. Usually, we have a catalogue, which contains coordinates (l, b) and proper motions $(\mu_l \cos b, \mu_b)$ of stars. General model of proper motions describes the Sun motion and the galactic rotation and may be represented as

$$\mu = \sum_{i=1}^{n} P_i \Phi_i(l, b) + \varepsilon, \qquad (1)$$

where

P_i are the parameters of the model to be determined,

ε is the noise and systematic trends, which are not included into the model.

Φ_i are the functional basis of the model.

The main condition of correct use of the LST is: "ε in the formula (1) is nothing else but the Gaussian noise". We forget this very often since the LST always does produce a result, but it may be

- Formal, non physical
- Unreliable due to large r.m.s.e
- Distorted by systematic trends

It means that we always have to remember two things:

- Does the model or its part correspond to observations?
- What is beyond the accepted model? Is the model complete?

2. Orthogonal Representation

We call equation (2) the **physical model** of proper motions where functions Φ_i form the functional basis of the model $\mathbf{\Phi}$. Let us consider the **formal model** of proper motions with another functional basis $\mathbf{K}$:

$$\mu = \sum_{j=1}^{m} C_j K_i(l, b). \tag{2}$$

In this equation

C_j are the coefficients of expansion,

K_j are the set of functions, which are *orthogonal* on the celestial sphere.

We have to find out the dependence between C_j and P_i. In some bases $\mathbf{K}$, we can obtain

$$P_i \sim C_{p_i} \sim C_{q_i} \sim C_{r_i} \sim \ldots \tag{3}$$

for each P_i. It means, that we may derive each parameter of the physical model more than once and, consequently, we can compare them. This is the *basic idea* of the method. The method allows to estimate whether the behavior of the observational material follows the functions of physical basis or not.

Good agreement between the values derived from several harmonics shows the correspondence of the model to observations.

Bad agreement can show:

- Adopted model does not correspond to observations
- There are unknown systematic motions which penetrate into expansion coefficients

In addition, the existence of unpredictable non-zero coefficients indicates that adopted model is not complete.

Concerning the basis $\mathbf{K}$, we have to choose such set of functions which being applied to the physical basis, produces the relations like (3). Usually, in stellar kinematics, the *spherical functions* are good.

3. Test of the Oort-Linblad Model

The Oort-Linblad's model is widely used for the analysis of proper motions. The general view of the equation for proper motions in the galactic longitude is

$$k\mu_l \cos b = \pi V_x \sin l - \pi V_y \cos l + A \cos b \cos 2l + B \cos b \tag{4}$$

where

V_x, V_y are the components of the Sun velocity,

A, B are the Oort's parameters,

π is a trigonometric parallax,

k = 47.4 is a scale factor to reduce arcsec/cy into km/s kps^{-1}

We select as formal basis **K** the spherical harmonics defined as

$$K_{n^2+2k+m-1}(l,b) = \begin{cases} L_{n0}(b) & k=0, m=1 \\ L_{nk}(b)\sin kl & k\neq 0, m=0 \\ L_{nk}(b)\cos kl & k\neq 0, m=1 \end{cases} \tag{5}$$

In this basis for the Oort-Linblad's model we find that the parameters under consideration are proportional to the coefficients of the expansion:

$$\begin{array}{ll} V_x & \sim C_2 \sim C_{10} \sim \dots \\ V_y & \sim C_3 \sim C_{11} \sim \dots \\ A & \sim C_8 \sim C_{20} \sim \dots \\ B & \sim C_0 \sim C_4 \sim C_{16} \sim \dots \end{array} \tag{6}$$

The factors of proportionality may be found either theoretically for uniform distribution of stars over the celestial sphere, or numerically if the distribution is not uniform.

In some models (for example, three-dimensional rotation of the Galaxy), several parameters can hit one and same harmonics. In this case, we have to use the linear combinations of these parameters instead of individual ones.

4. Numerical Results

The universal program was written that can do all these things automatically. This program was applied to investigate the samples from the Hipparcos catalogue.

We illustrate the program by two samples for nearby (75–125 ps) and distant stars (300–500 ps) from the main sequence in H-R diagram. Also, the program was tested on artificial data[1] to show the effect in ideal case. The traditional solution using LST is shown in the table, too. The missed data in the table denote that the F-test (Broshe 1966) rejected the value in this cell.

This table shows that only the Sun motion can be detected certainly in the proper motions. The galactic rotation is polluted by unknown systematic motions which distort the 4th coefficient, though the Least Squares technique produces the solution that seems to be very reliable.

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[1]The following values of parameters had been accepted for simulation: $V_x = 9$, $V_y = 15$ km/s; $A = 12$, $B = -12$ km/s kps^{-1}; standard deviation of the Gaussian noise was 2 mas/yr and the range of distances was from 300 to 500 ps.

Parameter		Simulation	75 – 125 pc	300 – 500 pc
V_x	2	7.8 ± 0.3	9.1 ± 0.5	9.1 ± 0.6
	10	8.3 ± 0.7	5.2 ± 1.3	9.8 ± 1.7
	LST	8.0 ± 0.0	8.9 ± 0.3	9.2 ± 0.3
V_y	3	14.9 ± 0.3	15.8 ± 0.5	15.1 ± 0.6
	11	15.2 ± 0.7	14.5 ± 1.3	21.0 ± 1.7
	LST	15.0 ± 0.0	15.9 ± 0.3	15.3 ± 0.3
A	8	11.7 ± 0.8	—	12.9 ± 2.1
	20	8.5 ± 4.6	—	—
	LST	12.3 ± 0.1	12.4 ± 3.8	12.9 ± 1.0
B	0	-12.3 ± 0.6	-11.9 ± 4.1	-10.4 ± 1.4
	4	-11.7 ± 1.8	$-33.0 \pm 12.$	-21.8 ± 4.3
	16	-20.0 ± 8.9	—	—
	LST	-12.1 ± 0.1	-13.7 ± 2.7	-12.9 ± 1.0

5. Conclusion

The first attepmt of the method apllication for stellar kinematics was made in two papers by Vityazev & Tsvetkov (1989a, 1989b). The basic principles of our method are described in the paper by Vityazev (1994) who proposed to use the spherical functions to derive the mutual rotation between two reference frames. Our method is an extension of this approach for kinematics. It may be easily modified for many purposes if one wishes to test a model for correspondence to observational data.

Acknowledgments. The author expresses gratitude to the LOC of the ADASS'98 conference for hospitality and financial support.

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