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Background Fluctuation Analysis from the Multiscale Entropy

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Abstract. The entropy measurement has been proposed for image restoration several years ago in astronomy. Different entropies have been used for this purpose. The multiscale entropy has the advantage to introduce a noise modeling, and the concept of information at different level of resolution. It can be used for tracking the presence of signal in a noisy data set. We show in this paper that it can also be used for background fluctuations analysis. A range of examples illustrates the results. We finally apply this method on X-ray simulations (XMM-EPIC) of diffuse large scale structure emission in order to discriminate different cosmological models.

1. Introduction

The concept of entropy has been introduced in astronomy for 40 years especially for deconvolution regularization. Many entropy definitions have been proposed in the past, but they do not take into account the spatial correlation of the image. Some authors tried a pyramidal approach to take a spatial correlation into account (Bontekoe, Koper, & Kester 1994), but this adds complexity. The correct way to do it is to compute the entropy in the wavelet space (Starck, Murtagh, & Gastaud 1998):

$$H(X) = \sum_{j=1}^{l} \sum_{k=1}^{N_j} h(w_{j,k}) \tag{1}$$

with $h(w_{j,k}) = -\ln p(w_{j,k})$. Here h stands for the entropy (or information) relative to a wavelet coefficient, and H for the multiscale entropy (MSE) of a signal or an image. For Gaussian noise, we get

$$H(X) = \sum_{j=1}^{l} \sum_{k=1}^{N_j} \frac{w_{j,k}^2}{2\sigma_j^2} \tag{2}$$

where σ_j is the noise at scale j. We see that the information is proportional to the energy of the wavelet coefficients. This definition of entropy verify the following criteria:

1. *The information in a flat signal is zero.*
2. *The amount of information in a signal is independent of the background.*

3. *The amount of information is dependent on the noise. A given signal Y ($Y = X + \mathrm{Noise}$) does not furnish the same information if the noise is high or small.*
4. *The entropy must work in the same way for a pixel which has a value $B + \epsilon$ (B being the background), and for a pixel which has a value $B - \epsilon$.*
5. *The amount of information is dependent on the spatial correlation in the signal. If a signal S presents large homogeneous features above the noise, it contains less information.*

This definition has been used with success for restoration (Starck & Murtagh 1999).

2. Multiscale Entropy applied to Background Fluctuation Analysis

Previous equation is used to compute an average over the pixels:

$$E(j) = \frac{1}{N} \sum_{k=1}^{N} h(w_j) \tag{3}$$

$E(j)$ gives the mean entropy at the scale j. From the mean entropy vector, we have statistical informations on each scale separately. Having a noise model, we are able to calculate (generally from simulations) the mean entropy vector $E_{\mathrm{noise}}(j)$ resulting from a pure noise. Then we define the normalized mean entropy vector by

$$E_n(j) = \frac{E(j)}{E_{\mathrm{noise}}(j)} \tag{4}$$

Now, this function of the scale can be used to analyse the background fluctuations.

2.1. Detection of sources

Five simulated images were created by adding 400, 200, 100, 50, 0 sources to a noisy image. The background is a Gaussian noise of amplitude 1. The sources are given by:

$$I(x, y) = I_0 \exp\left(\frac{(x-a)^2 + (y-b)^2}{2\sigma^2}\right) \tag{5}$$

with $I_0 = 1$ and $\sigma = 2$. Defining the signal to noise ratio (SNR) as the ratio between the standard deviation in the smallest box which contains at least 90% of the flux of the source, and the noise standard deviation, we have a SNR equal to 0.25: the sources are not detectable. Several tests of this non-detectability have been made: χ^2 test in direct space (i.e., is this pixel above the Gaussian noise?), correlation in direct space (i.e., is this sub-image a psf?), wavelet filter: the routine mrfilter (MR/1 software[1]). The sources are really hard to detect:

[1] `http://ourworld.compuserve.com/homepages/multires`

genuine sources are lost in false detections (tens against ten thousands). The mean entropy function of the scale has been computed for these images. The error is estimated by drawing ten times each image. Figure 1 clearly shows that the sources increase the entropy of the image. But it is obvious that the positions of these sources remain unknown.

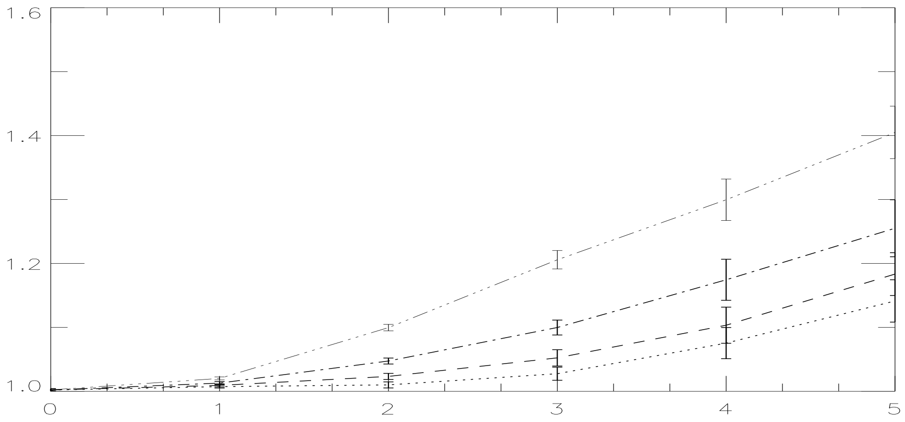

Figure 1. Mean entropy versus the scale of 5 simulated images containing undetect able sources and noise. Each curve corresponds to the multiscale of one image. From top to bottom, the image contains respectively 400, 200, 100, 50, and 0 sources, (which is confused with the x-axis).

2.2. Detection of faint feature

The new satellite XMM (to be launched in year 2000) will open a new era in the study of the sky at the energy from 0.1 to 10 keV. This will enable astronomers to detect large scale structures (hundreds of megaparsecs) and help to discriminate between different cosmological models. The problem is to detect the hot gas in filaments which are expected to be between galaxy clusters. The emission of these filaments, predicted from hydrodynamical simulation models, is very faint and diffuse. On the image of filaments will be superimposed very numerous background and foreground quasars and clusters of galaxies. We try the multiscale entropy method to detect the existence of filaments. Greg Bryan, from MIT, kindly provided simulation of the sky at different cosmological distances (redshift). From the density and the temperature cubes given by the simulations, a photons flux per simulation cell for the bandwidth from 0.4 keV to 4 keV is computed. The image of a filament located at $z = 0.5$ was computed with a pixel field of view of 4 arcsecond. The size of the image is 512×512 pixels. This image contains both clusters and filaments. In addition the point-like images of quasars are simulated using the law $\log(N)$–$\log(S)$ (Hasinger et al. 1998). The two images were summed, then multiplied by the integration time, here 400 kilosecond. Poisson noise, PSF blurring, vignetting are added. Two sets of images have been simulated, with and without the filaments. As previously, the mean entropy function of the scale has been computed. The difference between the two entropies, for the image with filament and without filament, is plotted in Fig. 2. The error bars e were estimated from from five realisations of the images, for both image with(σ_1) and without (σ_2) filaments

$$e = 2.5\sqrt{\sigma_1^2 + \sigma_2^2} \tag{6}$$

This difference is significant at the $2.5\,\sigma$ level (98.76%)

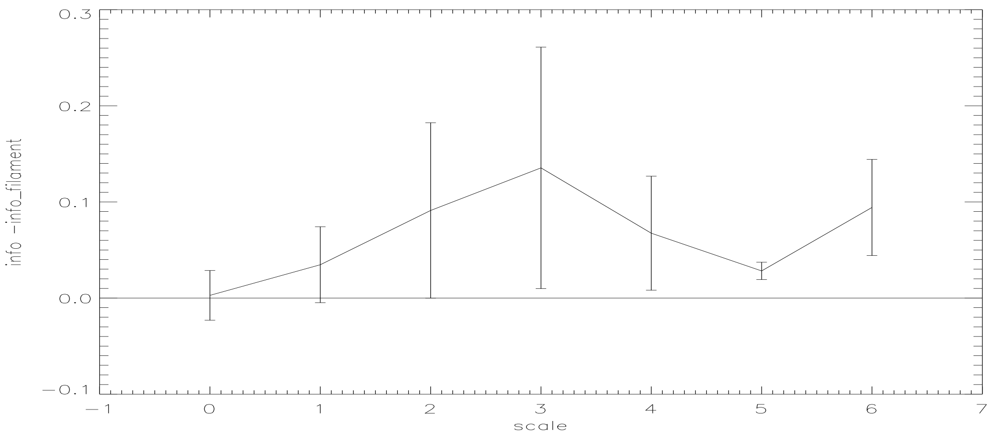

Figure 2. difference of mean entropy for the images with and without filament, error bar $2.5\,\sigma$.

3. Conclusion

We have seen that information must be measured from the transformed data, and not from the data itself. The transformation chosen was a non-orthogonal wavelet transform which is suited to astronomical images. The mean entropy is a good indicator of the presence of undetected sources, or faint extended structures. It is well suited to background fluctuations analysis.

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