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A New Entropy Measure Based on Multiple Resolution and Noise Modeling

Fionn Murtagh

Queen's University Belfast and Strasbourg Observatory

Jean-Luc Starck

CEA, DAPNIA, Saclay

Abstract. A new measure of information is defined which is based on noise modeling and incorporates resolution scale. Properties of this measure are discussed. Use of such an entropy measure for filtering and deconvolution is exemplified. Implications for structuring of, and access to, image archives are noted.

1. Introduction

We show how entropy can be incorporated into a multiscale image processing setting. The latter is a powerful setting for a wide range of image processing operations. Additionally it allows for image noise modeling, and subsequent noise removal.

Immediate applications include image filtering and deconvolution. Other objectives include object finding and definition, and feature characterization. More globally, we also note that such vision modeling will be necessary in future generation image databases.

Background on these methods can be found in Starck, Murtagh, & Bijaoui (1998). Information on a multiresolution image and vision software package, MR/1, can be found at `http://visitweb.com/multires`.

2. Multiscale Entropy Filtering Method

The multiscale entropy method consists of measuring the information h relative to wavelet coefficients, and of separating this into two parts h_s, and h_n. The expression h_s is called the signal information and represents the part of h which is definitely not contaminated by the noise. The expression h_n is called the noise information and represents the part of h which *may* be contaminated by the noise. We have $h = h_s + h_n$. Following this notation, the corrected (i.e., filtered) wavelet or multiscale coefficient $\tilde{w}$ should minimize:

$$J(\tilde{w}_j) = h_s(w_j - \tilde{w}_j) + \alpha h_n(\tilde{w}_j)$$

i.e. there is a minimum of information in the residual $(w - \tilde{w})$ due to the significant signal, and a minimum of information which can be due to the noise in the solution $\tilde{w}_j$.

In order to verify a number of properties, the following functions are proposed for h_s and h_n in the case of Gaussian noise:

$$
\begin{aligned}
h_s(w_j) &= \frac{1}{\sigma_j^2} \int_0^{|w_j|} u \text{ erf} \left(\frac{\mid w_j \mid - u}{\sqrt{2}\sigma_j} \right) du \\
h_n(w_j) &= \frac{1}{\sigma_j^2} \int_0^{|w_j|} u \text{ erfc} \left(\frac{\mid w_j \mid - u}{\sqrt{2}\sigma_j} \right) du
\end{aligned}
$$

3. Regularization Parameter and Data Model

The regularization parameter, α, can be determined by using the fact that we expect a residual with a given standard deviation at each scale j equal to the noise standard deviation σ_j at that scale. Hence we have an α_j per scale.

If we have a model, D_m, for the data we can use

$$J_m(\tilde{w}_j) = h_s(w_j - \tilde{w}_j) + \alpha h_n(\tilde{w}_j - w_m)$$

4. Regularized Entropy-Based Filtering

The regularized entropy-based filtering algorithm is as follows.

1. Estimate the noise in the data σ (see Olsen 1993; Starck & Murtagh 1998a).
2. Wavelet transform of the data.
3. Calculate from σ the noise standard deviation σ_j at each scale j.
4. Set $\alpha_j^{min} = 0$, $\alpha_j^{max} = 200$.
5. For each scale j do
   1. Set $\alpha_j = \frac{\alpha_j^{min} + \alpha_j^{max}}{2}$
   2. For each wavelet coefficient $w_{j,k}$ of scale j, find $\tilde{w}_{j,k}$ by minimizing $j_m(\tilde{w}_{j,k})$ or $j_{ms}(\tilde{w}_{j,k})$
   3. Calculate the standard deviation of the residual:

$$\sigma_j^r = \sqrt{\frac{1}{N_j} \sum_{k=1}^{N_j} (w_{j,k} - \tilde{w}_{j,k})^2}$$

   4. If $\sigma_j^r > \sigma_j$ then the regularization is too strong, and we set α_j^{max} to α_j, otherwise we set α_j^{min} to α_j.
5. If $\alpha_j^{max} - \alpha_j^{min} > \epsilon$ then goto 5.
6. Multiply all α_j by the constant α_u (default: 1).
7. For each scale j and for each wavelet coefficient w find $\tilde{w}_{j,k}$ by minimizing $j_m(\tilde{w}_{j,k})$ or $j_{ms}(\tilde{w}_{j,k})$.
8. Reconstruct the filtered image from $\tilde{w}_{j,k}$ by the inverse wavelet transform.

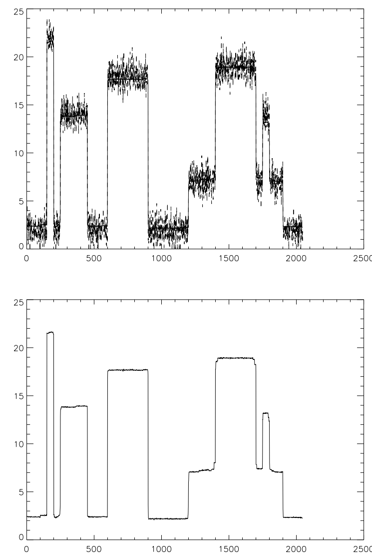

Figure 1. Top, noisy blocks and filtered blocks overplotted. Bottom, filtered blocks.

The minimization of j_m or j_{ms} (step 5.2) can be done by any method. For instance, a simple dichotomy can be used in order to find $\tilde{w}$ such that

$$\frac{\partial h_s(w - \tilde{w})}{\partial \tilde{w}} = -\alpha_j \frac{\partial h_n(\tilde{w})}{\partial \tilde{w}} \tag{1}$$

The idea to treat the wavelet coefficients such that the residual respects some constraint has also been used in Nason (1996) and Amato & Vuza (1998) using cross-validation.

5. Filtering: Examples

Figure 1 shows a difficult case of smooth and sharp transitions. The filtering method described here allows an excellent noise filtering of it to be carried out. Figure 2 shows a spectrum, and an effective noise filtered version.

6. Summary and Conclusion

We have described an integrated approach for specification of information content, handling of noise, and effective implementation. The multiscale entropy is similarly of benefit in deconvolution.

We note that large image repositories *require*

- Noise separation, for compression and quicklook
- Variable resolution over time, for progressive transmission, and variable resolution over space leading to multifoveated images.
- Usable approaches to information characterization.

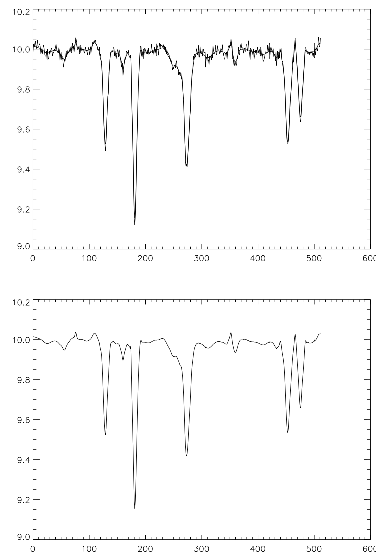

Figure 2. Top, real spectrum and filtered spectrum overplotted. Bottom, filtered spectrum.

Hence information- and entropy-based processing technologies will be necessary for the effective design and implementation of large, future generation, image databases.

Further reading is available in Starck, Murtagh, & Gastaud (1998) and Starck & Murtagh (1998b). The methods described here are available in the multiresolution analysis software package, MR/1, Version 2.0. Details of the MR/1 software package can be found at `http://visitweb.com/multires`.

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