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Generalized Linear Multi-Frequency Imaging in VLBI

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Abstract. In VLBI, generalized Linear Multi-Frequency Imaging (MFI) consists of multi-frequency synthesis (MFS) and multi-frequency analysis (MFA) of the VLBI data obtained from observations on various frequencies. A set of linear deconvolution MFI algorithms is described. The algorithms make it possible to obtain high quality images interpolated on any given frequency inside any given bandwidth, and to derive reliable estimates of spectral indexes for radio sources with continuum spectrum.

1. Statement of the problem

Let us consider a linear model for intensity $I_{kpq} = I(x_p, y_q, \nu_k)$ of the radio source in a point (x_p, y_q) on the observational frequency ν_k:

$$I_{kpq} \approx (I_0)_{pq} + (I_1)_{pq}\,\beta_k + \ldots + (I_{N-1})_{pq} \cdot (\beta_k)^{N-1},$$

$$\beta_k = \tfrac{\nu_k}{\nu_0} - 1, \quad k = 1,\ 2,\ \ldots\ ,\ K,$$

where ν_0 is reference frequency corresponding to the intensity $(I_0)_{pq}$.

If the intensity I_{kpq} in the point (x_p, y_q) can be approximated by power law as

$$I_{kpq} = (I_0)_{pq} \cdot \left(\frac{\nu_k}{\nu_0}\right)^{\alpha_{pq}},$$

then we can present it as

$$I_{kpq} = (I_0)_{pq}\, e^{\xi_k \alpha_{pq}} \approx (I_0)_{pq} \cdot (1 + \xi_k \alpha_{pq})$$

where $\xi_k = \ln\ (1 + \beta_k) \approx \beta_k$,

and thus the spectral indexes $\alpha_{pq} = \alpha\,(x_p, y_q)$ can be obtained as

$$(I_1)_{pq} = \alpha_{pq} \cdot (I_0)_{pq}$$

Let us consider a target function

$$\rho = \sum_{k=1}^{K} \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} w_{knm} \cdot \left| V_{knm} - \hat{V}_{knm} \right|^2,$$

where, $w_{knm} = w(u_n, v_m, \nu_k) \geq 0$ are weights, V_{knm}, $\hat{V}_{knm}$ is a measured and a model visibility function respectively,

$$\hat{V}_{knm} = A_k \cdot \sum_{p,q=o}^{M-1} \left[\sum_{l=o}^{N-1} \left(\widehat{I}_l \right)_{pq} \cdot \beta_k^l \right] \cdot \exp \left\{ -2\pi i \cdot (u_n x_p + v_m y_q) \right\},$$

where, A_k is a gain coefficient for k-th antenna,

$$\left(\widehat{I}_l \right)_{pq} = \Delta^2 \varphi_{pq} \cdot (I_l)_{pq} \left(1 - x_p^2 - y_q^2 \right)^{-0.5},$$

φ_{pq} is a normalized beam, Δ is a grid step.

The problem of the optimization can be presented as a solution of the following system of linear equations:

$$(D_0)_{pq} = 0, \ \ldots, \ (D_{N-1})_{pq} = 0$$

for a vector of intensity $\left(\widehat{\mathbf{I}} \right)_{rt} = \left(\left(\widehat{\mathbf{I}}_0 \right)_{rt}, \left(\widehat{\mathbf{I}}_1 \right)_{rt}, \ \ldots, \ \left(\widehat{\mathbf{I}}_{N-1} \right)_{rt} \right)^T$, where the m-th residual map $(D_m)_{pq}$ can be defined as:

$$(D_m)_{pq} = \sum_{k=1}^{K} \beta_k^m \cdot \left\{ D_{kpq} - \sum_{i=0}^{M-1} \sum_{l=0}^{M-1} B_{k,p-i,q-l} \cdot \sum_{n=0}^{N-1} \left(\widehat{\mathbf{I}}_n \right)_{il} \cdot \beta_k^n \right\}, \quad (1)$$

$$m = 0, \ 1, \ \ldots, \ N-1, \quad (2)$$

where, $D_{kpq} = \sum\limits_{n,m=o}^{M-1} w_{knm} \cdot V_{knm} \cdot \exp \left\{ 2\pi i \left(u_n x_p + v_m y_q \right) \right\}$ is a k-th "dirty" map at the point (x_p, y_q),

$B_{k,p-i,q-l} = \sum\limits_{n,m=o}^{M-1} w_{knm} \exp \left\{ 2\pi i \left[u_n \left(x_p - x_i \right) + v_m \left(y_q - y_l \right) \right] \right\}$ is a k-th "dirty" beam at the point $(x_p - x_i, y_q - y_l)$.

2. Solution of the problem

Let us choose the following initial conditions: $\left(\widehat{\mathbf{I}}_m \right)_{il}^{(0)} = 0$ for all m, i, l and form initial arrays $(D_m)_{pq}^{(0)}$, $m = 0, \ 1, \ \ldots, \ N-1$, and $\left(\widehat{B}_m \right)_{pq} = \sum\limits_{k=1}^{K} A_k^2 \cdot (\beta_k)^m \cdot B_{kpq}$, $m = 0, ..., 2N-2$.

Calculation of the next s-th step ($s = 1, \ 2, \ \ldots$) begins from the choice of the point (x_p, y_q), of the map maximum

$$\varepsilon^{(s-1)} = \max_{x_r^2 + y_t^2 < 1; \quad 0 \leq m < N} \left| (D_m)_{rt}^{(s-1)} \right|.$$

Now it's possible to specify a vector $\left(\widehat{\mathbf{I}}\right)_{pq}$:

$$\left(\widehat{\mathbf{I}}\right)_{pq}^{(s)} = \left(\widehat{\mathbf{I}}\right)_{pq}^{(s-1)} + \gamma \mathbf{E}^{-1} \cdot (\mathbf{D})_{pq}^{(s-1)},$$

and the residual maps $(\mathbf{D})_{rt} = \{(D_0)_{rt}, (D_1)_{rt}, ..., (D_{N-1})_{rt}\}^T$:

$$(\mathbf{D})_{rt}^{(s)} = (\mathbf{D})_{rt}^{(s-1)} - \widehat{\mathbf{B}}_{r-p,t-q} \cdot \left[\left(\widehat{\mathbf{I}}\right)_{pq}^{(s)} - \left(\widehat{\mathbf{I}}\right)_{pq}^{(s-1)}\right].$$

Here $\mathbf{E} = (E_{ij})$ is a positive defined matrix of maximum values of weighted "dirty" beams, $E_{ij} = \left(\widehat{B}_{i+j}\right)_{0,0}, i, j = 0, ..., N-1$; γ is a loop gain. The process of the iteration can be completed if $\varepsilon^{(s-1)} < \varepsilon$, where ε is a given accuracy. Otherwise it is necessary to suppose $s = s + 1$ and to calculate the next $\varepsilon^{(s)}$.Conditions of the convergence of the algorithm above is $0 < \gamma < 2$, $1 \leq N \leq K$.

The developed algorithm is nothing other than the *multi-frequency linear deconvolution*, itself. this is described in more detail this procedure by Likhachev, et al. (2003). Notice that the developed algorithm allows to synthesize and analyze of high-quality VLBI images directly from the visibility data measured on a few frequencies, without analyses of the images itself. In case of multi-frequency linear deconvolution, it is possible to synthesize an image of a radio source at any intermediate frequency *inside* any given frequency band. Thus, *spectral interpolation* of the image is feasible. This part of the algorithm is carry out the *synthesis* of the image itself. However, the algorithm also makes it possible to obtain an estimate of the *spectral index* for a given radio source, i.e., it implements the *analysis* of the image. It is clear that multi-frequency imaging (MFI) will provide the highest angular resolution possible for any VLBI project due to its improved (u, v)-coverage.

3. Implementation of the linear deconvolution algorithm

The algorithm described above was implemented in the software, ***Astro Space Locator (ASL) for Windows*** (http://platon.asc.rssi.ru/dpd/asl/asl.html). It was developed by the Laboratory for Mathematical Methods of the Astro Space Center (Likhachev, 2003).

Fig.1 shows two deconvolved images of 3C84 as observed on the VLBA at 11 and 15 GHz respectively. Due to the better (u,v)-coverage, the quality and angular resolution of the interpolated MFS-image at 11 GHz is much better than for the same source at 15 GHz.

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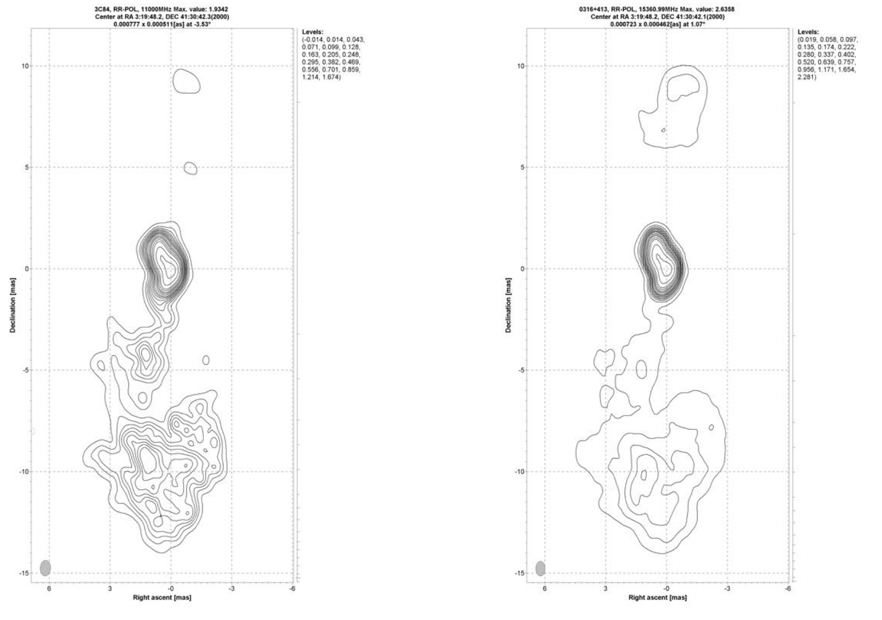

Figure 1. 3C84 observations at 11 (interpolated) and 15 GHz

References

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