

Phasor Algorithms of the SIM Fringe Estimation

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Abstract. The Space Interferometry Mission (SIM) will provide unprecedented micro-arcsecond (μas) precision to search for extra-solar planets and possible life in the universe. SIM will also revolutionize our understanding of the dynamics and evolutions of the local universe through hundred-fold improvements of inertial astrometry measurements. SIM has two so-called guide interferometers to provide stable inertial orientation knowledge of the baseline, and a science interferometer to measure target fringes. The guide and science measurements are based on the fringe phase measurements using a CCD detector. One of SIM's key issues is to develop a new algorithm for calculation of fringe parameters. Not only astrometric results need that new algorithm, but also real-time fringe tracking requires a new method to calculate phase and visibility fast and accurately. The formulas for the phasor algorithms for fringe estimation are presented. The signal-noise ratio performances of the fringe quadratures are demonstrated. The advantages of phasor algorithms for application of fast fringe tracking and on-board data compression are discussed.

1. Introduction

A space-based astrometric interferometry mission, SIM, is a long baseline optical interferometer optimized for global (inertial) and narrow-angle (relative) astrometry. Global astrometry with accuracy of 4 μas permit trigonometric determination of distances throughout the Galaxy. The narrow-angle astrometry at the level of 1 μas performs definite searches for extra-solar planets (Bahcall 1991). The SIM is the only astronomical instrument, that can unambiguously measure the masses of extra-solar planets. The SIM instrument has two guide interferometers and a science interferometer. The guide interferometers observe two bright objects in order to provide an inertially stable baseline. Delay measurements from guide stars are fed forward to the science interferometer. Based on the knowledge of the baseline orientation in space the science interferometer operates similar to the ground-based Michelson interferometers (Shao 1988). The key to interferometer measurements is the white light fringe determination. The SIM instrument uses path length modulation implemented on the optical delay line for fringe detection. All three interferometers in SIM use coherent fringe demodulation and active fringe tracking. The 500 Hz modulation uses a sawtooth waveform for 4 – 8 spectral channels simultaneously. The SIM in-

strument has broad wavelength coverage from ultraviolet to near-infrared. In order to reach high accuracy of fringe parameter measurements it is important to develop new phasor algorithms.

This paper presents formulas for fringe estimation based on the phasor algorithms, and demonstrate the signal-noise ratio performances of the fringe quadratures. The advantages of phasor algorithms for application of fast fringe tracking and on-board data compression are discussed.

2. Phasor Algorithm

SIM is a fringe-scanning interferometer that needs to acquire and track the white light fringe in order to equalize the paths of the two arms of the interferometer at the nanometer level. The pathlength modulation implemented by the voice coil uses a sawtooth waveform, and the detector is read out coherently using eight time bins per scan. The basic frame rate is 500 Hz, and each time bin is 0.125 ms. The path length in one arm varies linearly with the stroke, which has a length equal to the longest wavelength. Two guide-star interferometers have 4 spectral channels, and a science interferometer has 8 spectral channels. For all of spectral channels, one, or a little more than one fringe, is scanned across CCD detectors.

The fringe irradiance is written as :

$$F(\tau) = N/s * [1 + V \sin(k\tau - \phi)],$$

where N is the mean number of photons per scan, s is the stroke, V is fringe visibility, k is the wave number ($2\pi/\lambda$), and ϕ is the fringe phase. The modulation position $\tau = st/T - s/2$, where t is time for a bin, and T is the time interval of a stroke.

From the CCD measurements the accumulated photon counts for each time bin can be written as :

$$H_i = \int_{si/8}^{s(i+1)/8} F(\tau) d\tau, i = 1 - 8.$$

The total photons per stroke are the summation of counts in all eight time bins:

$$N_H = H_0 + H_1 + H_2 + H_3 + H_4 + H_5 + H_6 + H_7.$$

For each channel we calculate the X and Y quadratures for the dithered signal. In the simple case that the modulation amplitude matches a wavelength, i.e. $s = \lambda$, we can combine the 8 bin data as follows:

$$X' = -H_0 + H_2 - H_5 + H_7, \quad Y' = -H_1 + H_3 + H_4 - H_6.$$

For most of channels the stroke is longer than their effective wavelengths. We must define:

$$\begin{aligned} \theta &= \pi s / (4\lambda), & \text{where } s \text{ is stroke, } \lambda \text{ is wavelength;} \\ \alpha &= \sin \theta + \sin 2\theta - \sin 3\theta, \\ \beta &= \cos \theta - \cos 2\theta - \cos 3\theta + \cos 4\theta. \end{aligned}$$

The true quadratures, X_p & Y_p , i.e. X and Y phasor components, and the total fluxes are calculated as :

$$\begin{aligned} X_p &= \pi s / (\sqrt{2}\lambda) * [X' / \beta - Y' / \alpha], \\ Y_p &= \pi s / (\sqrt{2}\lambda) * [X' / \beta + Y' / \alpha] \\ N_p &= N_H - \sin 4\theta * Y' / \alpha \end{aligned}$$

So the fringe visibility and phase are calculated as follows:

$$\begin{aligned} V^2 &= (\langle X_p \rangle^2 + \langle Y_p \rangle^2) / \langle N_p \rangle^2 ; \\ \phi &= \arctan(\langle Y_p \rangle / \langle X_p \rangle) - \pi/4, \end{aligned}$$

where angle brackets $\langle \dots \rangle$ represent an average over certain time periods of fringe tracking.

The photon counts of time bins obey Poisson distribution. The signal-noise ratio of phasors can be expressed as

$$\begin{aligned} SNR_X &= X_p / \sqrt{X_p + N_d + N_r}; \\ SNR_Y &= Y_p / \sqrt{Y_p + N_d + N_r}; \end{aligned}$$

where N_d is the dark current, and N_r is the read noises.

Simulations are conducted for a typical case of 30 seconds of fringe tracking. For a 7th magnitude guide stars the signal-noise ratios of X, Y phasors are shown in Figure 1. For comparison, the signal-noise ratio of phasors in X and Y directions are computed for the case of four time bins (Colavita 1999). It is shown that the performances of signal-noise ratio drops significantly when the wavelength of a spectral channel is shorter than the length of the stroke. It is necessary to use new eight time bin algorithm for uniform and improved performance.

3. Discussion

This work developed new phasor algorithm and has been successfully used for the MAM testbed experiments (Shaklan 1992). In order to keep the paths of the two arms nearly equal, the MAM delay line must acquire and track the white light fringe phase. Once the fringe visibility calculated by the formulas above exceeds a threshold, fringe tracking can start. By using the phase results from the algorithm the system reports the absolute value of the fringe phase as telemetry at the maximum rate that the fringe control loop runs (Hines 2002). For such real-time control system the phasor algorithm has proven to be simple, fast and accurate.

Traditional non-linear fitting techniques, or the pseudo-inverse method are much too slow, and are difficult to use for nanometer and millisecond control. The phasor algorithm also can be used for the on-board data processing in SIM. The data volumes that must be down-loaded from the spacecraft to the ground station are extremely high. Phasors computed by this algorithm can be used to compress fringe data. It is important to maximize measurement information while reducing data volume in SIM.

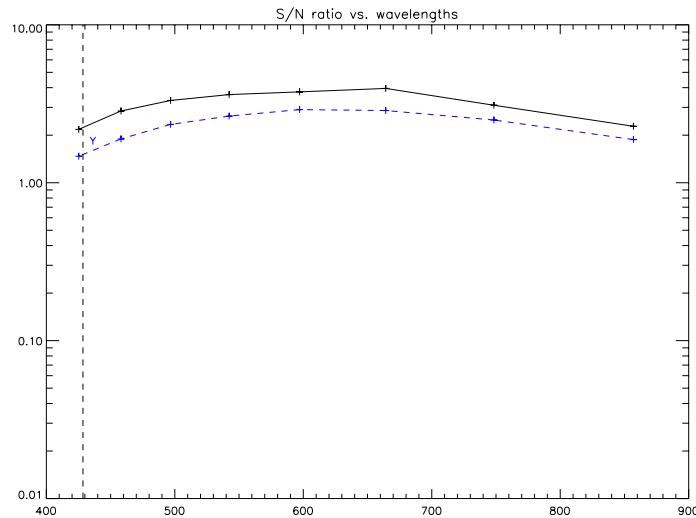


Figure 1. Signal-noise ratios of X, Y phasors

This paper presents the preliminary study of the phasor algorithm. Cyclic errors, the vibration effects, and nonlinear strokes are a few example of noises that will reduce the accuracy of fringe parameter determination. Those issues need to be addressed in the near future.

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