Astronomical Data Analysis Software and Systems XII
ASP Conference Series, Vol. 295, 2003
H. E. Payne, R. I. Jedrzejewski, and R. N. Hook, eds.

Solving for Polarization Leakage in Radio Interferometers Using Unpolarized Source

S. Bhatnagar,[1] R. V. Urvashi[2] and R. Nityananda[3]

National Center for Radio Astrophysics (NCRA), Pune, India 411007

Abstract. This paper presents an algorithm for solving antenna based polarization leakage in a radio interferometer using co-polar observations of unpolarized sources. If ignored, polarization leakage manifests itself as closure errors in parallel hand (co-polar) visibilities. Many working radio telescopes offer observational advantages for observations in non-polar mode (e.g., higher frequency resolution, lower integration time, etc.). Many observations are therefore done in non-polar mode and the computation of antenna based leakage gains in co-polar visibilities is scientifically useful for debugging and calibrating the instrument. Also, this is a useful option to use when one cannot or does not want to put in the additional measurement effort to determine the leakage term by means of cross-polar measurements. We also present results from test data taken with the Giant Meterwave Radio Telescope (GMRT) and discuss the degeneracy in the solutions and the equivalence of the leakage induced closure phase and the Pancharatnam phase of optics.

1. Introduction

Co-polar output of an interferometer, can be written as

$$\rho_{ij}^{pp} = \langle (g_i^p E_{i,\circ}^p + \alpha_i^q E_{i,\circ}^q + \epsilon_i)(g_j^p E_{j,\circ}^p + \alpha_j^q E_{j,\circ}^q + \epsilon_j)^\star \rangle \quad (1)$$

where p and q are two orthogonal polarization states (R and L or X and Y), g_i^p the antenna complex gain for the p-channel of antenna i, α_i^q the leakage of q-signal into the p-channel, $E_{i,\circ}^p$ the *ideal* response of the p-channel to the incident radiation, and ϵ_i the antenna based additive noise. For an unpolarized point source, $\langle E_{i,\circ}^p E_{j,\circ}^{q^\star} \rangle = \langle E_{i,\circ}^q E_{j,\circ}^{p^\star} \rangle = 0$ and $\langle E_{i,\circ}^p E_{j,\circ}^{p^\star} \rangle = \langle E_{i,\circ}^q E_{j,\circ}^{q^\star} \rangle = \rho_{ij,\circ}^{pp} = I/2$ where I is the total intensity. Writing $X_{ij}^{pp} = \rho_{ij}^{pp} / \rho_{ij,\circ}^{pp}$ we get

$$X_{ij}^{pp} = g_i^p g_j^{p^\star} + \alpha_i^q \alpha_j^{q^\star} + \epsilon_{ij} \quad (2)$$

---

[1]Now at the National Radio Astronomy Observatory (NRAO), Socorro, NM 87801, Email: sbhatnag@aoc.nrao.edu

[2]Birla Institute of Technology & Science (BITS), Pilani, India and now the department of Computer Science and Engineering, UCSD, CA 92093. Email: uraovenk@cs.ucsd.edu

[3]Email: rajaram@ncra.tifr.res.in

ϵ_{ij} is the independent baseline based noise. Usually this represents the noise in ρ_{ij}^{pp} after the correlation operation plus the antenna based noise. ϵ_{ij} therefore is a measure of the *true* closure errors in the system and is usually small. Assuming α_i^qs to be negligible, the usual Selfcal algorithm estimates g_i^ps such that $\sum_{\substack{i,j \\ i\neq j}} \left| X_{ij}^{pp} - g_i^p g_j^{p^\star} \right|^2$ is minimized. However, leakage due to mechanical and/or electronic imperfections in the feed, cross talk, squint of *cross-polar* primary beam, off-axis primary beam polarization, etc., is hard to eliminate making α_i^qs potentially non-negligible.

In the presence of significant α_i^qs (compared to $\sqrt{\epsilon_{ij}}$), ignoring the second term in Equation 2 will be equivalent to a system with *apparent* increased closure noise ($\epsilon_{ij} + \alpha_i^q \alpha_i^{q^\star}$ instead of just ϵ_{ij}). *Hence, polarization leakage manifests as increased closure errors.* This has also been pointed out by Rogers (1983) in the context of VLBA observations, and extensive study by Massi & Aaron (1997) for EVN shows that imaging quality is limited by these errors.

2. Algorithm

When solving for only g_i^p using co-polar visibilities, the α_i^qs appear as increased closure noise and will result in non-optimal solutions. Hence, a simultaneous solution for g_i^p and α_i^q would be optimal. In the presence of significant polarization leakage, the correct estimator for the *true* closure noise is given by $S = \sum_{\substack{i,j \\ i\neq j}} \left| X_{ij}^{pp} - (g_i^p g_j^{p^\star} + \alpha_i^q \alpha_j^{q^\star}) \right|^2 w_{ij}^{pp}$ where w_{ij}^{pp} are the weights. Equating the partial derivatives $\partial S/\partial g_i^{p\star}$, $\partial S/\partial \alpha_i^{q\star}$ to zero, we get a set of non-linear equations for g_i^ps and α_i^qs which can be iteratively solved (Bhatnagar & Nityananda 2001).

3. Solution Degeneracy, Simulations and the GMRT Experiment

Simulations demonstrate that with the use of the above algorithm, the χ^2 remains constant with increasing leakage, and that it solves for α_i^qs only if they are significant (i.e., distinguishable from $\sqrt{\epsilon_{ij}}$, Bhatnagar & Nityananda 2001). The decrease in χ^2, compared to that given by Selfcal, is due to the use of the correct estimator for the closure noise and not because of extra free parameters (the α_i^qs) in the problem. *The solutions for α_i^qs are therefore physically meaningful.*

However, an obvious degeneracy is rotation of all the g's by a common phase factor and the αs by an, in general different, phase factor, does not affect the left hand side of Equation 2. We also have the freedom to choose a suitable basis in polarization space (see Bhatnagar & Nityananda 2001 for details). We choose this basis in such a way that the sum of the absolute squares of all the leakage terms is minimized. Carrying out the maximization of $\sum |g_i|^2$ by the method of Lagrange multipliers, subject to a constant χ^2, we obtain the condition that $\sum \alpha_i^* g_i = 0$ (implying that the leakage coefficients be orthogonal to the gains) and can be incorporated by first choosing an overall phase for the α's so that $\sum \alpha_i^\star g_i$ is real. Then, carry out a rotation in the $g - \alpha$ plane by an angle θ satisfying $\tan\theta = \sum \alpha_i^\star g_i / (\sum (g_i g_i^\star - \alpha_i \alpha_i^\star)$. Results of such a

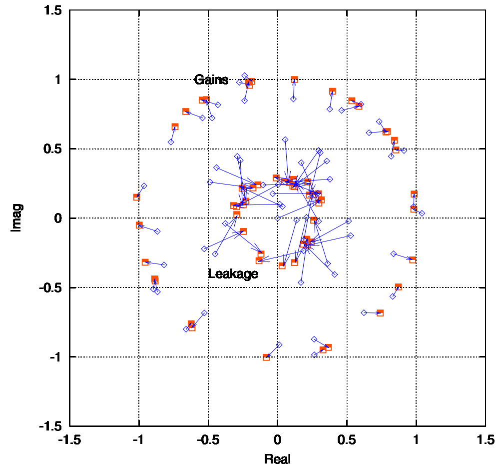

Figure 1. Simulations showing the decoupling of the solutions for g_i^ps and α_i^qs. Squares are the input test g_i^ps and α_i^qs on the complex plane while diamonds are the solutions. The arrows show the length and direction of correction due to the transformation.

transform on simulated data are shown in Figure 1. The absolute frame of reference in which α_i^qs are measured, also remain undetermined since the source is unpolarized. However this degeneracy is same as that in the phase of g_i^ps and is not important for correcting the data. We used GMRT L-band test data with circularly polarized feed on only one antenna (`C03` in Figure 2) and linear feeds on the rest. In the mean linear basis of all the antennas, `C03` appears as an antenna with $\alpha_i^q = \alpha_i^p \sim 1$ ($E_{C03}^R = E_{C03}^X e^{-\iota\delta} + E_{C03}^Y e^{\iota\delta}$; ideally $\delta = \pi/4$). The fractional leakage (α_i^q/g_i^p) for all antennas is plotted in the complex plane in Figure 2[Left]. Mean leakage of all the antennas define the reference frame in which the leakage of the nominally linear antennas is minimum. All but one nominally linearly polarized antennas are at the origin (minimal leakage); points corresponding to `C03` are farthest from the center, grouped $\sim 180°$ apart.

4. Poincaré Sphere and the Pancharatnam Phase

A general elliptically polarized state can be written as a superposition of two states represented by the vector $[\cos\theta/2 \quad \sin\theta/2 e^{\iota\phi}]$ in the basis defined by the left- and right-circular polarization states. Clearly, $\theta = \pi/2$ corresponds to linear polarization and $\theta \neq 0, \pi/2$ to elliptical polarization. The Poincaré sphere representation of the state of polarization maps the general elliptic state to the point (θ, ϕ) on the sphere. It can be shown that the closure phase between three coherent, non-identical, antennas (points I,J and K in Figure 2[Right]) is equal to half the solid angle IJK. This goes by the name Pancharatnam/Geometric/Berry's phase in optical literature (Pancharatnam, 1956). *The closure phase due to polarization mis-matches between phased antennas in a radio interferometer therefore naturally measures the Pancharatnam phase.*

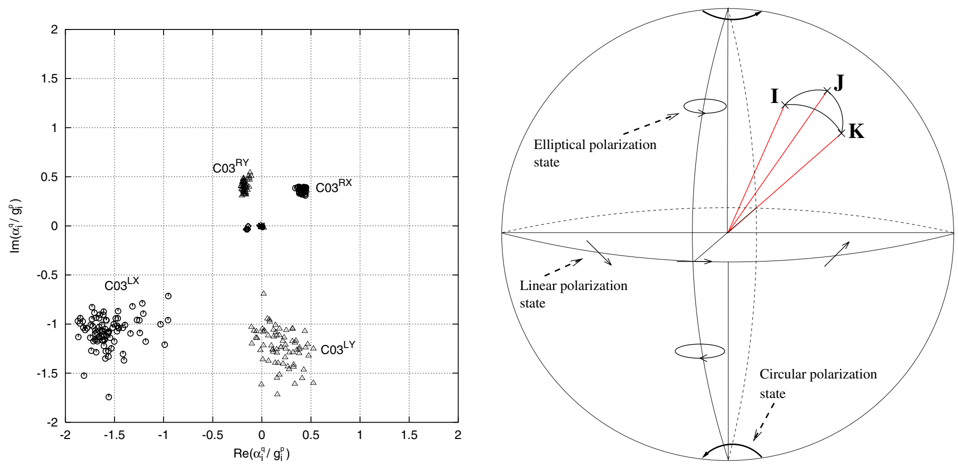

Figure 2. **[Left]** Plot of α_i^q/g_i^p for all GMRT antennas. $C03^{LX}$ and $C03^{RX}$ (open circles) are from correlations of R- and L-channel of C03 with X-channel of other linear antennas. Similarly for $C03^{LY}$ and $C03^{RY}$ (triangles). One of the linearly polarized antennas is leakier than the others and L-channel of C03 is noisier than its R-channel. **[Right]** Poincaré sphere representation of polarization states. Closure phase between three coherent but non-identical antennas represented by the points I,J and K is equal to half the solid angle of IJK.

5. Conclusions

The method described here measures the polarization leakage using the *co-polar* visibilities for an unpolarized calibrator. It is a useful tool for studying the polarization purity of the antennas of radio interferometers. Simultaneous solution of gain and leakage ensures that the method factorizes for leakage only if they are distinguishable from intrinsic *true* closure noise. The degeneracy between solutions of antenna based complex gains and leakage is broken by physically meaningful transform and the solutions can be used to remove leakage induced closure errors. Geometric interpretation of the results on the Poincaré sphere shows that the leakage induced closure phase is same as the Pancharatnam phase and the degeneracy in the solutions can be understood as a rigid rotation of the Poincaré sphere.

Acknowledgments. We thank the GMRT staff, NCRA, BITS and NRAO for their support and co-operation.

References

Bhatnagar, S. & Nityananda, R. 2001, A&A, 375, 344-350

Massi, M. & Aaron, S. 1997, EVN Tech. Memo, N75

Pancharatnam, S. 1956, S. Proc. Indian Aad. Sci., A44, 247

Rogers, A. E. E. 1983, VLB Array Memo No. 253